

Flavor Mixing Democracy and Minimal CP Violation

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Abstract

We point out that there is a unique parametrization of quark flavor mixing in which every angle is close to the Cabibbo angle $\theta_C \simeq 13^\circ$ with the CP-violating phase ϕ_q around 1° , implying that they might all be related to the strong hierarchy among quark masses. Applying the same parametrization to lepton flavor mixing, we find that all three mixing angles are comparably large (around $\pi/4$) and the Dirac CP-violating phase ϕ_l is also minimal as compared with its values in the other eight possible parametrizations. In this spirit, we propose a simple neutrino mixing ansatz which is equivalent to the tri-bimaximal flavor mixing pattern in the $\phi_l \rightarrow 0$ limit and predicts $\sin \theta_{13} = 1/\sqrt{2} \sin(\phi_l/2)$ for reactor antineutrino oscillations. Hence the Jarlskog invariant of leptonic CP violation $J_l = (\sin \phi_l)/12$ can reach a few percent if θ_{13} lies in the range $7^\circ \leq \theta_{13} \leq 10^\circ$.

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1 Introduction

Within the standard electroweak model, the origin of CP violation is attributed to an irremovable phase of the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) quark flavor mixing matrix [1] in the charged-current interactions:

$$-\mathcal{L}_{\text{cc}}^q = \frac{g}{\sqrt{2}} \overline{(u \ c \ t)}_{\text{L}} \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{L}} W_\mu^+ + \text{h.c.} . \quad (1)$$

The size of this nontrivial CP-violating phase depends on the explicit parametrization of the CKM matrix V . One may in general describe V in terms of three rotation angles and one CP-violating phase, and arrive at nine topologically different parametrizations [2]. If V takes the Cabibbo flavor mixing pattern [3]

$$V_{\text{C}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} , \quad (2)$$

where $\omega = e^{i2\pi/3}$ is the complex cube-root of unity (i.e., $\omega^3 = 1$), then one can immediately find that the CP-violating phases in all the nine parametrizations are exactly $\pi/2$. Hence V_{C} characterizes the case of “maximal CP violation” in a parametrization-independent way, although it is not a realistic quark flavor mixing matrix. Among the nine parametrizations of V listed in Ref. [2], the one advocated by the Particle Data Group [4] is most popular and its CP-violating phase is about 65° . The idea of a “geometrical T violation” has been suggested in Ref. [5] to explain such a CP-violating phase around $\pi/3$. In comparison, the CP-violating phase is about 90° in the parametrization recommended in Ref. [6] or in the original Kobayashi-Maskawa representation [7]. Accordingly, the concept of “maximal CP violation” has sometimes been used to refer to a quark flavor mixing scenario in which the CP-violating phase equals $\pi/2$ for given values of the mixing angles [8, 9, 10, 11].

Of course, the value of the CP-violating phase is correlated with the values of the mixing angles in a given parametrization of V . Indeed, the parametrization itself depends on the chosen flavor basis and only the moduli of the matrix elements V_{ij} are completely basis-independent. Although all the parametrizations of V are mathematically equivalent, one of them might be phenomenologically more interesting in the sense that it might either make the underlying physics of quark mass generation and CP violation more transparent or lead to more straightforward and simpler relations between the fundamental flavor mixing parameters and the corresponding observable quantities. It is therefore meaningful to examine different parametrizations of the CKM matrix V and single out the one which is not only phenomenologically useful but also allows us to have a new insight into the flavor puzzles and possible solutions to them.

In this paper we pose such a question: is it possible to ascribe small CP-violating effects in the quark sector to a strongly suppressed CP-violating phase in the CKM matrix V in which all three mixing angles are comparably sizable? The answer to this question is actually affirmative as already observed in Ref. [12], and the details of such a nontrivial description of quark flavor mixing and CP violation will be elaborated in section 2. We show that the CP-violating phase ϕ_q is only about 1° , while every quark mixing angle is close to the Cabibbo angle $\theta_C \simeq 13^\circ$ in this unique parametrization of V , implying that they might all have something to do with the strong hierarchy of quark masses. We argue that this particular representation reveals an approximate flavor mixing democracy and “minimal CP violation”. It also provides a simple description of the structure of the matrix V , which is almost symmetric in modulus about its V_{ud} - V_{cs} - V_{tb} axis.

Applying the same parametrization to the lepton flavor mixing, we find that all three angles are comparably large (around $\pi/4$) and the Dirac CP-violating phase ϕ_l is also minimal as compared with its values in the other eight possible parametrizations. We start from this observation to propose a simple and testable neutrino mixing ansatz which is equal to the well-known tri-bimaximal flavor mixing pattern [13] in the $\phi_l \rightarrow 0$ limit. It predicts $\sin \theta_{13} = 1/\sqrt{2} \sin(\phi_l/2)$ for reactor antineutrino oscillations, and its two larger mixing angles are consistent with solar and atmospheric neutrino oscillations. The Jarlskog invariant for leptonic CP violation turns out to be $J_l = (\sin \phi_l)/12$, which can reach a few percent if θ_{13} lies in the range $7^\circ \leq \theta_{13} \leq 10^\circ$.

2 Quark flavor mixing

The parametrization of the CKM matrix V , which assures an approximate flavor mixing democracy and nearly minimal CP violation in the quark sector, takes the form

$$\begin{aligned}
 V &= \begin{pmatrix} c_y & 0 & s_y \\ 0 & 1 & 0 \\ -s_y & 0 & c_y \end{pmatrix} \begin{pmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & e^{-i\phi_q} \end{pmatrix} \begin{pmatrix} c_z & 0 & -s_z \\ 0 & 1 & 0 \\ s_z & 0 & c_z \end{pmatrix} \\
 &= \begin{pmatrix} c_x c_y c_z + s_y s_z e^{-i\phi_q} & s_x c_y & -c_x c_y s_z + s_y c_z e^{-i\phi_q} \\ -s_x c_z & c_x & s_x s_z \\ -c_x s_y c_z + c_y s_z e^{-i\phi_q} & -s_x s_y & c_x s_y s_z + c_y c_z e^{-i\phi_q} \end{pmatrix}, \tag{3}
 \end{aligned}$$

where $c_x \equiv \cos \theta_x$ and $s_x \equiv \sin \theta_x$, and so on. Without loss of generality, we arrange the mixing angles to lie in the first quadrant but allow the CP-violating phase ϕ_q to vary between zero and 2π . Comparing Eq. (1) with Eq. (3), we immediately arrive at the relation $\cos \theta_x = |V_{cs}|$ together with

$$\tan \theta_y = \left| \frac{V_{ts}}{V_{us}} \right|,$$

$$\tan \theta_z = \left| \frac{V_{cb}}{V_{cd}} \right|. \quad (4)$$

In this parametrization the off-diagonal asymmetries of V in modulus [14] are given as

$$\begin{aligned} \Delta_L^q &\equiv |V_{us}|^2 - |V_{cd}|^2 = |V_{cb}|^2 - |V_{ts}|^2 = |V_{td}|^2 - |V_{ub}|^2 = s_x^2 (s_z^2 - s_y^2), \\ \Delta_R^q &\equiv |V_{us}|^2 - |V_{cb}|^2 = |V_{cd}|^2 - |V_{ts}|^2 = |V_{tb}|^2 - |V_{ud}|^2 = s_x^2 (c_y^2 - s_z^2). \end{aligned} \quad (5)$$

Note that the other eight parametrizations listed in Ref. [2] are unable to express Δ_L^q and Δ_R^q in such a simple way. Furthermore, the Jarlskog invariant for CP violation [15] reads

$$J_q = \text{Im} (V_{ud} V_{cs} V_{us}^* V_{cd}^*) = \text{Im} (V_{us} V_{cb} V_{ub}^* V_{cs}^*) = \cdots = c_x s_x^2 c_y s_y c_z s_z \sin \phi_q. \quad (6)$$

We observe that choosing $|V_{cs}|$, Δ_L^q , Δ_R^q and J_q as four independent parameters to describe the CKM matrix V is also an interesting possibility, because they determine the geometric structure of V and its CP violation in a straightforward and rephasing-invariant manner.

To see the point that θ_x , θ_y and θ_z are comparable in magnitude, let us express them in terms of the well-known Wolfenstein parameters [16]. Up to the accuracy of $\mathcal{O}(\lambda^6)$, the Wolfenstein-like expansion of the CKM matrix V [17] is given as

$$V \simeq \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda \left[1 - A^2\lambda^4 \left(\frac{1}{2} - \rho \right) + iA^2\lambda^4\eta \right] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}(1 + 4A^2)\lambda^4 & A\lambda^2 \\ A\lambda^3 \left[1 - \left(1 - \frac{1}{2}\lambda^2 \right) (\rho + i\eta) \right] & -A\lambda^2 \left[1 - \lambda^2 \left(\frac{1}{2} - \rho \right) + i\lambda^2\eta \right] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}, \quad (7)$$

where $\lambda = 0.2253 \pm 0.0007$, $A = 0.808_{-0.015}^{+0.022}$, $\rho = 0.135_{-0.014}^{+0.023}$ and $\eta = 0.350 \pm 0.013$ extracted from a global fit of current experimental data on flavor mixing and CP violation in the quark sector [4]. Comparing Eq. (7) with Eq. (3), we arrive at the approximate relations

$$\begin{aligned} \tan \theta_x &\simeq \lambda \left[1 + \frac{1}{2} (1 + A^2) \lambda^2 \right], \\ \tan \theta_y &\simeq A\lambda \left[1 - \frac{1}{2} (1 - 2\rho) \lambda^2 \right], \\ \tan \theta_z &\simeq A\lambda, \\ \sin \phi_q &\simeq \lambda^2 \eta \left[1 + \frac{1}{2} (1 + 2A^2 - 2\rho) \lambda^2 \right], \end{aligned} \quad (8)$$

which hold up to the accuracy of $\mathcal{O}(\lambda^5)$. Therefore, we obtain

$$\theta_x \simeq 13.2^\circ, \quad \theta_y \simeq 10.1^\circ, \quad \theta_z \simeq 10.3^\circ, \quad \phi_q \simeq 1.1^\circ. \quad (9)$$

We see that the small difference between θ_y and θ_z signifies a slight off-diagonal asymmetry of the CKM matrix V in modulus about its V_{ud} - V_{cs} - V_{tb} axis. Note that this tiny asymmetry is quite stable against the renormalization-group-equation (RGE) running effects from the electroweak scale to a superhigh-energy scale or vice versa. Indeed, only the Wolfenstein parameter A is sensitive to the RGE evolution [18] so that θ_y and θ_z run in the same way

even at the two-loop level ¹. In contrast, θ_x and ϕ_q are almost insensitive to the RGE running effects. The striking fact that the CP-violating phase ϕ_q is especially small in this parametrization was first emphasized in Ref. [12]. Indeed, the other eight parametrizations listed in Ref. [2] all require $\phi_q \geq 60^\circ$. Moreover, the values

$$\Delta_L^q \simeq 6.3 \cdot 10^{-5}, \quad \Delta_R^q \simeq 4.9 \cdot 10^{-2}, \quad J_q \simeq 3.0 \cdot 10^{-5} \quad (10)$$

indicate that $\theta_y = \theta_z$ and $\phi_q = 0$ might be two good leading-order approximations from the point of view of model building. In these two limits the CKM matrix V is real and symmetric in modulus. Consequently, the small off-diagonal asymmetry and the small CP-violating phase of V might come from some complex perturbations at the level of quark mass matrices.

Why may $\phi_q \sim \lambda^2$ coexist with $\theta_x \sim \theta_y \sim \theta_z \sim \lambda$? The reason is simply that V_{ub} is the smallest CKM matrix element and only a small ϕ_q guarantees a significant cancellation in $V_{ub} = -c_x c_y s_z + s_y c_z e^{-i\phi_q}$ to make $|V_{ub}| \sim \mathcal{O}(\lambda^4)$ hold ². The point that V_{ub} strongly depends on ϕ_q motivates us to propose a phenomenological ansatz for quark flavor mixing in which $V_{ub} \rightarrow 0$ holds in the $\phi_q \rightarrow 0$ limit. In this case we find that the condition $\tan \theta_y = \tan \theta_z \cos \theta_x$ must be fulfilled and the CKM matrix reads

$$V_0 = \begin{pmatrix} s_y/s_z & s_x c_y & 0 \\ -s_x c_z & c_x & s_x s_z \\ s_x^2 c_y s_z & -s_x s_y & c_z/c_y \end{pmatrix}. \quad (11)$$

Of course, V_0 can approximately describe the observed moduli of the nine CKM matrix elements. The relation $\tan \theta_y = \tan \theta_z \cos \theta_x$ implies that θ_z must be slightly larger than θ_y , and thus it has no conflict with the numerical results obtained in Eq. (9). Now the CP-violating phase ϕ_q is switched on and V_0 is changed to

$$V = \begin{pmatrix} (c_z^2 + s_z^2 e^{-i\phi_q}) s_y/s_z & s_x c_y & -s_y c_z (1 - e^{-i\phi_q}) \\ -s_x c_z & c_x & s_x s_z \\ -c_y s_z (c_x^2 - e^{-i\phi_q}) & -s_x s_y & (s_y^2 + c_y^2 e^{-i\phi_q}) c_z/c_y \end{pmatrix}, \quad (12)$$

which predicts $|V_{ub}| = 2s_y c_z \sin(\phi_q/2) \simeq s_y c_z \sin \phi_q$ for very small ϕ_q . Comparing Eq. (12) with Eq. (7), we arrive at $\tan \theta_x \simeq \lambda$, $\tan \theta_y \simeq \tan \theta_z \simeq A\lambda$ and $\sin \phi_q \simeq \lambda^2 \sqrt{\rho^2 + \eta^2}$ in the leading-order approximation. We conclude that this ansatz is essentially valid, and it provides us with a good lesson for dealing with lepton flavor mixing in section 3.

It has long been speculated that the small quark flavor mixing angles might be directly related to the strong quark mass hierarchies [19, 20], in particular when the quark mass

¹We thank H. Zhang for confirming this point using the two-loop RGEs of gauge and Yukawa couplings.

²Because of $A \simeq 0.808$, $\rho \simeq 0.135$ and $\eta \simeq 0.350$, the true order of $|V_{ub}|$ is λ^4 instead of λ^3 . Following the original spirit of the Wolfenstein parametrization [16], one may consider to take $V_{ub} = A\lambda^4 (\hat{\rho} - i\hat{\eta})$ by redefining two $\mathcal{O}(1)$ parameters $\hat{\rho} = \rho/\lambda \simeq 0.599$ and $\hat{\eta} = \eta/\lambda \simeq 1.553$.

matrices possess a few texture zeros which can naturally originate from a certain flavor symmetry [21]. In this sense it is also interesting for us to consider possibly simple and instructive relations between quark mass ratios (m_u/m_c , m_c/m_t , m_d/m_s and m_s/m_b) and flavor mixing parameters (θ_x , θ_y , θ_z and ϕ_q) in the parametrization of V under discussion. In view of the values for the quark masses renormalized at the electroweak scale [22], we make the naive conjectures

$$\begin{aligned}\sin \theta_x &\simeq \sqrt{\frac{m_d}{m_s} + \frac{m_u}{m_c}} , \\ \sin \theta_y &\simeq \sin \theta_z \simeq \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} , \\ \sin \phi_q &\simeq \frac{m_s}{m_b} .\end{aligned}\tag{13}$$

Of course, these approximate relations are only valid at the electroweak scale, and whether they can easily be derived from a realistic model of quark mass matrices remains an open question. But a possible correlation between the smallness of the CP-violating phase and the smallness of quark mass ratios (e.g., $\sin \phi_q \simeq m_s/m_b$ as first conjectured in Ref. [12]) is certainly interesting and suggestive, because it might imply a common origin for the quark mass spectrum, flavor mixing and CP violation. We hope that such a phenomenological observation based on our particular parametrization in Eq. (3) may be useful to infer the presence of an underlying flavor symmetry from the experimental data in the near future.

3 Lepton flavor mixing

We proceed to consider the 3×3 Maki-Nakagawa-Sakata-Pontecorvo (MNSP) lepton flavor mixing matrix [23] in the weak charged-current interactions:

$$-\mathcal{L}_{cc}^l = \frac{g}{\sqrt{2}} \overline{(e \ \mu \ \tau)}_L \gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L W_\mu^- + \text{h.c.} .\tag{14}$$

The MNSP matrix U can be parametrized in the same way as in Eq. (3):

$$\begin{aligned}U &= \begin{pmatrix} c_b & 0 & s_b \\ 0 & 1 & 0 \\ -s_b & 0 & c_b \end{pmatrix} \begin{pmatrix} c_a & s_a & 0 \\ -s_a & c_a & 0 \\ 0 & 0 & e^{-i\phi_l} \end{pmatrix} \begin{pmatrix} c_c & 0 & -s_c \\ 0 & 1 & 0 \\ s_c & 0 & c_c \end{pmatrix} P_\nu \\ &= \begin{pmatrix} c_a c_b c_c + s_b s_c e^{-i\phi_l} & s_a c_b & -c_a c_b s_c + s_b c_c e^{-i\phi_l} \\ -s_a c_c & c_a & s_a s_c \\ -c_a s_b c_c + c_b s_c e^{-i\phi_l} & -s_a s_b & c_a s_b s_c + c_b c_c e^{-i\phi_l} \end{pmatrix} P_\nu ,\end{aligned}\tag{15}$$

where $P_\nu = \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ denotes an irremovable phase matrix if the massive neutrinos are Majorana particles, $c_a \equiv \cos \theta_a$ and $s_a \equiv \sin \theta_a$, and so on. Current experimental data

indicate that at least two lepton mixing angles are much larger than the Cabibbo angle $\theta_C \simeq 13^\circ$ [4]. In particular, the tri-bimaximal flavor mixing pattern [13]

$$U_0 = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P_\nu \quad (16)$$

is quite consistent with the observed values for the solar and atmospheric neutrino mixing angles and can easily be derived from a number of neutrino mass models based on discrete flavor symmetries [24]. Comparing Eq. (15) with Eq. (16), we see that they become equivalent to each other if the conditions

$$\theta_a \simeq 54.7^\circ, \quad \theta_b = 45^\circ, \quad \theta_c = 60^\circ, \quad \phi_l = 0^\circ \quad (17)$$

are satisfied. A particularly interesting point is that the relation $\tan \theta_b = \tan \theta_c \cos \theta_a$ exactly holds and thus the matrix element $U_{e3} = -c_a c_b s_c + s_b c_c e^{-i\phi_l}$ automatically vanishes as ϕ_l approaches zero. This observation, together with the promising ansatz for the quark flavor mixing discussed in Eqs. (11) and (12), motivates us to consider the following lepton flavor mixing ansatz:

$$U = \begin{pmatrix} \frac{1}{2\sqrt{6}} (1 + 3e^{-i\phi_l}) & \frac{1}{\sqrt{3}} & -\frac{1}{2\sqrt{2}} (1 - e^{-i\phi_l}) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{6}} (1 - 3e^{-i\phi_l}) & -\frac{1}{\sqrt{3}} & \frac{1}{2\sqrt{2}} (1 + e^{-i\phi_l}) \end{pmatrix} P_\nu, \quad (18)$$

which reproduces the tri-bimaximal flavor mixing pattern U_0 in the $\phi_l \rightarrow 0$ limit. In other words, the generation of nonzero U_{e3} is directly correlated with the nonzero CP-violating phase ϕ_l (or vice versa). Similar to the case of quark flavor mixing, all three lepton mixing angles are comparably large in this parametrization. Hence it also assures the “minimal CP violation” in the lepton sector, although one has not yet observed CP-violating effects in neutrino oscillations.

One may similarly calculate the Jarlskog invariant of leptonic CP violation and off-diagonal asymmetries of U in modulus based on Eqs. (15) and (18). The results are

$$J_l = \text{Im} (U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*) = \text{Im} (U_{e2} U_{\mu 3} U_{e3}^* U_{\mu 2}^*) = \cdots = c_a s_a^2 c_b s_b c_c s_c \sin \phi_l = \frac{1}{12} \sin \phi_l, \quad (19)$$

and

$$\begin{aligned} \Delta_L^l &= |U_{e2}|^2 - |U_{\mu 1}|^2 = |U_{\mu 3}|^2 - |U_{\tau 2}|^2 = |U_{\tau 1}|^2 - |U_{e3}|^2 = s_a^2 (s_c^2 - s_b^2) = +\frac{1}{6}, \\ \Delta_R^l &= |U_{e2}|^2 - |U_{\mu 3}|^2 = |U_{\mu 1}|^2 - |U_{\tau 2}|^2 = |U_{\tau 3}|^2 - |U_{e1}|^2 = s_a^2 (c_b^2 - s_c^2) = -\frac{1}{6}. \end{aligned} \quad (20)$$

It becomes obvious that the MNSP matrix U is more asymmetric in modulus than the CKM matrix V , and CP violation in the lepton sector is likely to be much larger than that in the quark sector simply because the lepton flavor mixing angles are not suppressed.

To see why the ansatz proposed in Eq. (18) is phenomenologically interesting in a more direct way, let us compare it with the standard parametrization of the MNSP matrix U [4]. In this case the neutrino mixing angles are predicted to be

$$\begin{aligned}\sin \theta_{13} &= \frac{1}{\sqrt{2}} \sin \frac{\phi_l}{2}, \\ \tan \theta_{12} &= \frac{1}{\sqrt{2 - 3 \sin^2 \theta_{13}}}, \\ \tan \theta_{23} &= \frac{1}{\sqrt{1 - 2 \sin^2 \theta_{13}}}.\end{aligned}\tag{21}$$

So $\theta_{13} \leq 45^\circ$ must hold for arbitrary values of ϕ_l . Given the generous experimental upper bound $\theta_{13} < \theta_C$ [4], the upper limit of ϕ_l turns out to be $\phi_l < 37.1^\circ$. A global analysis of current neutrino oscillation data seems to favor $\theta_{13} \simeq 8^\circ$ [25], implying $\phi_l \simeq 22.7^\circ$ together with $\theta_{12} \simeq 35.7^\circ$ and $\theta_{23} \simeq 45.6^\circ$. These results are certainly consistent with the present experimental data. The resulting value of the leptonic Jarlskog parameter is $J_l = (\sin \phi_l)/12 \simeq 3.2\%$, which should be large enough to be observed in the future long-baseline neutrino oscillation experiments. Furthermore, the CP-violating phase δ_l in the standard parametrization is found to be much larger than ϕ_l in our ansatz:

$$\sin \delta_l = \frac{\sqrt{2} \cos^2 \theta_{13}}{\sqrt{2 - 3 \sin^2 \theta_{13}}}.\tag{22}$$

Therefore, we obtain $\delta_l \simeq 84.4^\circ$ for $\theta_{13} \simeq 8^\circ$. Note again that $\theta_{13} \leq 45^\circ$ holds, so Eq. (22) is always valid for the experimentally allowed range of θ_{13} .

The values of the charged-lepton masses at the electroweak scale have already been given in Ref. [22], from which we obtain $m_e/m_\mu \simeq 4.74 \cdot 10^{-3}$ and $m_\mu/m_\tau \simeq 5.88 \cdot 10^{-2}$. In view of the neutrino mass-squared differences extracted from current neutrino oscillation experiments [25], we get $m_2/m_3 \simeq 0.17$ in the $m_1 \simeq 0$ limit for a normal mass hierarchy. A naive conjecture is therefore

$$\sin \phi_l \simeq \sqrt{\frac{m_2}{m_3}},\tag{23}$$

implying $\phi_l \simeq 24.3^\circ$ and thus $\theta_{13} \simeq 8.6^\circ$. Since θ_a , θ_b and θ_c are all large, it seems more difficult to link them to the charged-lepton or neutrino mass ratios.

Finally, it is worth pointing out that one may propose similar ansätze of lepton flavor mixing based on some other constant patterns with $U_{e3} = 0$. For example, we find that the mixing angles of the democratic [26], bimaximal [27], golden-ratio [28] and hexagonal [29] mixing patterns expressed in our present parametrization can also satisfy the condition $\tan \theta_b = \tan \theta_c \cos \theta_a$, and thus the matrix element $U_{e3} = -c_a c_b s_c + s_b c_c e^{-i\phi_l}$ automatically vanishes in the $\phi_l \rightarrow 0$ limit. Given such a constant pattern, a lepton flavor mixing ansatz

analogous to the one proposed in Eq. (18) can similarly be discussed. Its salient feature is therefore the prediction

$$\sin \theta_{13} = |U_{e3}| = 2s_b c_c \sin \frac{\phi_l}{2}, \quad (24)$$

which directly links ϕ_l to θ_{13} . Given $\theta_b = 45^\circ$ and $\theta_c = 60^\circ$ for the tri-bimaximal flavor mixing pattern, the first relation in Eq. (21) can then be reproduced from Eq. (24).

4 Summary

We have explored a unique parametrization of fermion flavor mixing in which the mixing angles are nearly democratic and the (Dirac) CP-violating phase is minimal. Within such a parametrization of the CKM matrix V we have shown that all three quark mixing angles are close to the Cabibbo angle $\theta_C \simeq 13^\circ$ while the CP-violating phase ϕ_q is only about 1° . It also provides a simple description of the structure of V , which is almost symmetric in modulus about its V_{ud} - V_{cs} - V_{tb} axis. When the MNSP matrix U is parametrized in the same way, we find that the lepton mixing angles are comparably large (around $\pi/4$) and the Dirac CP-violating phase ϕ_l is also minimal as compared with its values in the other eight possible parametrizations. These interesting observations have motivated us to propose a simple and testable neutrino mixing ansatz which is equal to the well-known tri-bimaximal flavor mixing pattern in the $\phi_l \rightarrow 0$ limit. It predicts $\sin \theta_{13} = 1/\sqrt{2} \sin(\phi_l/2)$ for reactor antineutrino oscillations, and its two larger mixing angles are consistent with solar and atmospheric neutrino oscillations. The Jarlskog invariant of leptonic CP violation is found to be $J_l = (\sin \phi_l)/12$, which can reach a few percent if θ_{13} lies in the range $7^\circ \leq \theta_{13} \leq 10^\circ$.

It is worth remarking that the unique parametrization discussed in this paper provides us with a novel description of the observed phenomena of quark and lepton flavor mixings. Different from other possible parametrizations suggesting either a “geometrical” or a “maximal” CP-violating phase, it allows us to deal with a “minimal” one. Although it remains unclear whether such a new point of view is really useful in our quest for the underlying flavor dynamics of fermion mass generation and CP violation, we believe that it can at least help understanding the structure of flavor mixing in a phenomenologically interesting way.

Note added: Soon after this paper appeared in the preprint archive (arXiv:1203.0496), the Daya Bay Collaboration announced their first $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillation result: $\sin^2 2\theta_{13} = 0.092 \pm 0.016(\text{stat}) \pm 0.005(\text{syst})$ (or equivalently, $\theta_{13} \simeq 8.8^\circ \pm 0.8^\circ$) at the 5.2σ level [30]. We find that our expectations, such as $\theta_{13} \simeq 8.6^\circ$ given below Eq. (23), are in good agreement with the Daya Bay observation.

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